Determination of *h*/*e* from the Photoelectric Effect

Appendices

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Appendix A: Weighted Least Squares Fitting

Developed from Taylor's discourse[16], the following derives the determination of the slope of a linear fit to data where each dependent variable, y_i, have different errors, σ_i . Assuming no error in the independent variable, x_i, the probability of obtaining y_i is

$$Prob_{A,B}(y_i) \propto \frac{1}{\sigma_i} e^{-(y_i - A - Bx_i)^2/2\sigma_i^2}$$

The probability of obtaining the set of y_i's is then

$$\begin{aligned} Prob_{A,B}(y_1, \ldots, y_N) &= Prob_{A,B}(y_1) \cdots Prob_{A,B}(y_N) \\ &\propto \frac{1}{\sigma_1} e^{-(y_1 - A - Bx_1)^2/2\sigma_1^2} \cdots \frac{1}{\sigma_N} e^{-(y_N - A - Bx_N)^2/2\sigma_N^2} \\ &= \frac{1}{\sigma_1 \cdots \sigma_N} e^{-\chi^2/2} \end{aligned}$$

where $\chi^2 = \sum_{i=1}^{N} \frac{(y_i - A - Bx_i)^2}{\sigma_i^2}$. The best fit would have χ^2 minimized by setting its partial derivatives with respect to A and B equal to zero and solving,

$$\begin{aligned} \frac{\partial \chi^2}{\partial A} &= -2 \sum_{i=1}^{N} \frac{1}{\sigma_i^2} (y_i - A - Bx_i) = 0 \\ \frac{\partial \chi^2}{\partial B} &= -2 \sum_{i=1}^{N} \frac{x_i}{\sigma_i^2} (y_i - A - Bx_i) = 0 \end{aligned}$$

where $w_i = \frac{1}{\sigma_i^2}$ to give

$$A = \frac{\sum wx^2 \sum wy - \sum wx \sum wxy}{\Delta} = \frac{\sum \left[S_{wxx} w_i - S_{wx} w_i x_i \right] y_i}{\Delta} \qquad \text{(A.1)}$$

$$B = \frac{\sum w \sum wxy - \sum wx \sum wy}{\Delta} = \frac{\sum \left[S_w w_i x_i - S_{wx} w_i \right] y_i}{\Delta} \qquad \text{(A.2)}$$

where $\Delta = \sum w \sum wx^2 - \left(\sum wx \right)^2$, $S_w = \sum w$, $S_{wx} = \sum wx$, $S_{wxx} = \sum wx^2$.

Using the method of quadrature for error propagation, the error in A and B can be found.

$$
\begin{aligned}
\delta A &= \sqrt{\sum\left(\frac{\partial A}{\partial y_i}\delta A_i\right)^2} = \sqrt{\sum\left(\frac{\partial A}{\partial y_i}\right)^2 \frac{1}{w_i}} \\
&= \sqrt{\sum\left(\frac{S_{wxx} w_i - S_{wx} w_i x_i}{\Delta}\right)^2 \frac{1}{w_i}} \\
&= \sqrt{\frac{1}{\Delta^2}\sum\left(S_{wxx}^2 w_i + S_{wx}^2 w_i x_i^2 - 2 S_{wxx} S_{wx} w_i x_i\right)} \\
&= \sqrt{\frac{1}{\Delta^2}\left(S_{wxx}^2 S_w + S_w^2 S_{wxx} - 2 S_{wxx} S_{wx}^2\right)} \\
&= \sqrt{\frac{S_{wxx}}{\Delta^2}\left(S_w S_{wxx} - S_{wx}^2\right)} = \sqrt{\frac{\sum wx^2}{\Delta^2}\Delta} \\
&= \sqrt{\frac{\sum wx^2}{\Delta}} \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
\delta B &= \sqrt{\sum\left(\frac{\partial B}{\partial y_i}\delta y_i\right)^2} = \sqrt{\sum\left(\frac{\partial B}{\partial y_i}\right)^2 \frac{1}{w_i}} \\
&= \sqrt{\sum\left(\frac{S_w w_i x_i - S_{wx} w_i}{\Delta}\right)^2 \frac{1}{w_i}} \\
&= \sqrt{\frac{1}{\Delta^2}\sum\left(S_w^2 w_i x_i^2 + S_{wx}^2 w_i - 2 S_w S_{wx} w_i x_i\right)} \\
&= \sqrt{\frac{1}{\Delta^2}\left(S_w^2 S_{wxx} + S_{wx}^2 S_w - 2 S_w S_{wx}^2\right)} \\
&= \sqrt{\frac{S_w}{\Delta^2}\left(S_w S_{wxx} - S_{wx}^2\right)} = \sqrt{\frac{\sum w}{\Delta^2}\Delta} \\
&= \sqrt{\frac{\sum w}{\Delta}} \qquad \text{(A.4)}
\end{aligned}
$$

According to Taylor[?], if the uncertainty in x is significant, then Δy should be modified to

$$
\Delta y_{\text{equiv}} = \sqrt{(\Delta y)^2 + \left(\frac{dy}{dx}\Delta x\right)^2} \qquad \text{(A.5)}
$$

For the linear regression case, $dy/dx = B$. Substituting Δy_{equiv} for Δy in the previous derivation would then give the correct treatment. All linear fits in the experiment follow this approach.

Appendix B: Data Collected on 18th and 20th November 2005

Yellow Filter

V (V)	ΔV (V)	I (nA)	ΔI (nA)	I' (nA)	I0 (+/-0.3nA)	Date
20.10	0.01	1019	0.1	100.5	100.5	20th Nov 05
19.03	0.01	1017	0.1	100.3	100.5	20th Nov 05
18.00	0.01	1015	0.2	100.1	100.5	20th Nov 05
17.04	0.01	1013	0.1	99.9	100.5	20th Nov 05
16.01	0.01	1012	0.1	99.8	100.5	20th Nov 05
15.07	0.01	1011	0.1	99.7	100.5	20th Nov 05
14.00	0.01	1010	0.1	99.6	100.5	20th Nov 05
13.05	0.01	1008	0.1	99.4	100.5	20th Nov 05
12.01	0.01	1004	0.1	99.0	100.5	20th Nov 05
11.00	0.01	998	0.1	98.4	100.5	20th Nov 05
10.09	0.01	995	0.1	98.1	100.5	20th Nov 05
9.04	0.01	990	0.1	97.6	100.5	20th Nov 05
8.00	0.01	984	0.1	97.0	100.5	20th Nov 05
7.01	0.01	975	0.1	96.2	100.5	20th Nov 05
6.02	0.01	969	0.1	95.6	100.5	20th Nov 05
4.97	0.01	960	0.1	94.7	100.5	20th Nov 05
3.98	0.01	950	0.1	93.7	100.5	20th Nov 05
3.069	0.001	935	0.1	92.2	100.5	20th Nov 05
2.507	0.001	925	0.1	91.2	100.5	20th Nov 05
2.245	0.001	919	0.1	90.6	100.5	20th Nov 05
2.011	0.001	908	0.1	89.6	100.5	20th Nov 05
1.747	0.001	899	0.1	88.7	100.5	20th Nov 05
1.506	0.001	888	0.1	87.6	100.5	20th Nov 05
1.256	0.001	870	0.1	85.8	100.5	20th Nov 05
1.002	0.001	849	0.1	83.7	100.5	20th Nov 05
0.900	0.001	836	0.1	82.5	100.5	20th Nov 05
0.802	0.001	825	0.1	81.4	100.5	20th Nov 05
0.707	0.001	812	0.1	80.1	100.5	20th Nov 05
0.602	0.001	794	0.1	78.3	100.5	20th Nov 05
0.502	0.001	773	0.1	76.2	100.5	20th Nov 05
0.399	0.001	752	0.1	74.2	100.5	20th Nov 05
0.302	0.001	726	0.1	71.6	100.5	20th Nov 05
0.197	0.001	690	0.1	68.1	100.5	20th Nov 05
0.109	0.001	653	0.1	64.4	100.5	20th Nov 05
0.023	0.001	600	0.1	59.2	100.5	20th Nov 05
0.000	0.001	581	0.1	57.3	100.5	20th Nov 05
-0.004	0.001	579	0.1	57.1	100.5	20th Nov 05
-0.008	0.001	575	0.1	56.7	100.5	20th Nov 05
-0.015	0.001	570	0.1	56.2	100.5	20th Nov 05
-0.020	0.001	564	0.1	55.6	100.5	20th Nov 05
-0.022	0.001	563	0.1	55.5	100.5	20th Nov 05

Only for the yellow filter is there a need for a different maximum forward current, I_0, therefore the original measured I' and rescaled I value

-0.639	0.001	-9.51	0.001	-0.598	100.5	20th Nov 05
-0.662	0.001	-9.67	0.001	-0.649	100.5	20th Nov 05
-0.682	0.001	-9.99	0.001	-0.962	100.5	20th Nov 05
-0.692	0.001	-10.82	0.001	-1.708	100.5	20th Nov 05
-0.569	0.001	-10.67	0.001	-1.620	100.5	20th Nov 05
-0.578	0.001	-10.66	0.001	-1.550	100.5	20th Nov 05
-0.608	0.001	-10.92	0.001	-1.378	100.5	20th Nov 05
-0.625	0.001	-11.18	0.001	-1.192	100.5	20th Nov 05
-0.652	0.001	-11.06	0.001	-1.016	100.5	20th Nov 05
-0.676	0.001	-11.92	0.001	-1.327	100.5	20th Nov 05
-0.699	0.001	-11.69	0.001	-1.630	100.5	20th Nov 05
-0.727	0.001	-11.55	0.001	-1.636	100.5	20th Nov 05
-0.743	0.001	-11.56	0.001	-1.446	100.5	20th Nov 05
-0.879	0.001	-11.66	0.001	-1.650	100.5	20th Nov 05
-0.899	0.001	-11.76	0.002	-1.160	100.5	20th Nov 05
-0.905	0.001	-11.98	0.001	-1.272	100.5	20th Nov 05
-0.906	0.001	-12.03	0.001	-1.286	100.5	20th Nov 05
-0.920	0.001	-12.46	0.001	-1.499	100.5	20th Nov 05
-0.238	0.001	-12.20	0.005	-1.213	100.5	20th Nov 05
-0.202	0.001	-12.60	0.005	-1.243	100.5	20th Nov 05
-0.256	0.001	-12.79	0.005	-1.267	100.5	20th Nov 05
-2.266	0.001	-13.62	0.005	-1.287	100.5	20th Nov 05
-2.279	0.001	-13.47	0.005	-1.319	100.5	20th Nov 05
-0.282	0.001	-13.26	0.005	-1.342	100.5	20th Nov 05
-0.290	0.001	-13.02	0.002	-1.365	100.5	20th Nov 05
-0.305	0.001	-19.58	0.002	-19.379	100.5	20th Nov 05
-0.399	0.001	-18.12	0.001	-18.402	100.5	20th Nov 05
-0.609	0.001	-17.58	0.001	-17.538	100.6	20th Nov 05
-0.329	0.001	-16.74	0.001	-16.606	100.6	20th Nov 05
-0.308	0.001	-15.96	0.001	-15.629	100.6	20th Nov 05
-0.360	0.001	-15.06	0.001	-13.697	100.6	20th Nov 05
-0.369	0.001	-15.35	0.005	-13.650	100.6	20th Nov 05
-0.370	0.001	-12.67	0.002	-12.663	100.6	20th Nov 05
-0.399	0.001	-15.39	0.002	-1.692	100.6	20th Nov 05
-0.390	0.001	-15.42	0.002	-1.690	100.6	20th Nov 05
-0.402	0.001	-16.29	0.002	-1.698	100.6	20th Nov 05
-0.402	0.001	-15.62	0.002	-0.760	100.6	20th Nov 05
-0.409	0.001	-15.62	0.002	-0.762	100.6	20th Nov 05
-0.429	0.001	-15.84	0.001	-0.779	100.6	20th Nov 05
-0.402	0.001	-15.72	0.002	-0.701	100.6	20th Nov 05
-0.460	0.001	-12.58	0.001	-0.249	100.6	20th Nov 05
-0.465	0.001	-15.83	0.001	-0.725	100.6	20th Nov 05
-0.470	0.001	-5.44	0.001	-0.537	100.5	20th Nov 05
-0.478	0.001	-6.31	0.001	-0.622	100.5	20th Nov 05
-0.491	0.001	-7.48	0.001	-0.738	100.5	20th Nov 05
-0.500	0.001	-8.13	0.001	-0.802	100.5	20th Nov 05
-0.510	0.001	-8.72	0.001	-0.860	100.5	20th Nov 05

Green Filter

V (V)	ΔV (V)	I (nA)	ΔI (nA)	Date
20.10	0.01	1019	2	18th Nov 05
19.03	0.01	1018	2	20th Nov 05
18.03	0.01	1015	2	18th Nov 05
17.04	0.01	1014	2	20th Nov 05
16.00	0.01	1012	2	20th Nov 05
15.01	0.01	1010	2	18th Nov 05
14.01	0.01	1008	2	20th Nov 05
13.03	0.01	1004	2	20th Nov 05
12.04	0.01	999	2	20th Nov 05
11.01	0.01	993	1	20th Nov 05
10.02	0.01	992	2	18th Nov 05
9.03	0.01	986	1	20th Nov 05
8.00	0.01	979	1	20th Nov 05
7.00	0.01	971	1	20th Nov 05
6.00	0.01	963	1	20th Nov 05
5.01	0.01	951	2	18th Nov 05
4.07	0.01	945	1	20th Nov 05
3.04	0.01	926	1	20th Nov 05
2.500	0.001	912	1	20th Nov 05
2.000	0.001	892	1	20th Nov 05
1.748	0.001	883	1	20th Nov 05
1.501	0.001	866	1	20th Nov 05
1.253	0.001	846	1	20th Nov 05
1.007	0.001	813	1	20th Nov 05
0.875	0.001	802	1	20th Nov 05
0.749	0.001	779	1	20th Nov 05
0.622	0.001	755	1	20th Nov 05
0.500	0.001	726	1	20th Nov 05
0.373	0.001	690	1	20th Nov 05
0.252	0.001	649	1	20th Nov 05
0.147	0.001	603	1	20th Nov 05
0.023	0.001	527	1	20th Nov 05
0.000	0.001	506	1	20th Nov 05
-0.022	0.001	490	0.5	20th Nov 05
-0.091	0.001	418	1	20th Nov 05
-0.114	0.001	388	1	20th Nov 05
-0.136	0.001	359	0.5	20th Nov 05
-0.149	0.001	342	0.5	20th Nov 05
-0.161	0.001	327	1	20th Nov 05
-0.179	0.001	303	1	20th Nov 05
-0.203	0.001	257	1	18th Nov 05
-0.221	0.001	229	1	18th Nov 05
-0.240	0.001	200	1	18th Nov 05
-0.262	0.001	169	0.5	18th Nov 05

-0.282	0.001	-13.52	0.01	20th Nov 05
-0.304	0.001	-13.65	0.01	20th Nov 05
-0.322	0.001	-13.73	0.01	20th Nov 05
-0.340	0.001	-13.87	0.02	20th Nov 05
-0.363	0.001	-13.96	0.01	20th Nov 05
-0.381	0.001	-14.03	0.01	20th Nov 05
-0.405	0.001	-14.15	0.01	20th Nov 05
-0.423	0.001	-14.24	0.05	20th Nov 05
-0.442	0.001	-14.32	0.02	20th Nov 05
-0.460	0.001	-14.62	0.01	20th Nov 05
-0.479	0.001	6.39	0.01	20th Nov 05
-0.502	0.001	1.446	0.002	20th Nov 05
-0.520	0.001	1.506	0.001	20th Nov 05
-0.540	0.001	-4.00	0.005	20th Nov 05
-0.561	0.001	-5.82	0.005	20th Nov 05
-0.582	0.001	-7.15	0.01	20th Nov 05
-0.601	0.001	-8.04	0.01	20th Nov 05
-0.620	0.001	-8.56	0.01	20th Nov 05
-0.642	0.001	-9.04	0.01	20th Nov 05
-0.660	0.001	-9.31	0.01	20th Nov 05
-0.682	0.001	-9.54	0.01	20th Nov 05
-0.702	0.001	-9.70	0.01	20th Nov 05
-0.726	0.001	-9.82	0.01	20th Nov 05
-0.748	0.001	-9.93	0.01	20th Nov 05
-0.774	0.001	-10.02	0.01	20th Nov 05
-0.799	0.001	-10.08	0.01	20th Nov 05
-0.826	0.001	-10.27	0.01	20th Nov 05
-0.852	0.001	-10.34	0.01	20th Nov 05
-0.900	0.001	-10.45	0.01	20th Nov 05
-0.951	0.001	-10.50	0.02	20th Nov 05
-1.005	0.001	-10.58	0.01	20th Nov 05
-1.104	0.001	-10.71	0.01	20th Nov 05
-1.252	0.001	-10.90	0.01	20th Nov 05
-1.509	0.001	-11.16	0.01	20th Nov 05
-1.754	0.001	-11.36	0.01	20th Nov 05
-2.006	0.001	-11.55	0.01	20th Nov 05
-2.516	0.001	-11.85	0.01	20th Nov 05
-3.005	0.001	-12.05	0.01	20th Nov 05
-3.56	0.001	-12.23	0.01	20th Nov 05
-4.01	0.01	-12.35	0.01	20th Nov 05
-4.54	0.01	-12.43	0.01	20th Nov 05
-5.01	0.01	-12.55	0.01	20th Nov 05
-6.03	0.01	-12.77	0.01	20th Nov 05
-7.03	0.01	-13.01	0.01	20th Nov 05
-8.05	0.01	-13.13	0.01	20th Nov 05
-9.05	0.01	-13.28	0.01	20th Nov 05
-10.05	0.01	-13.41	0.01	20th Nov 05

Blue Filter

V (V)	ΔV (V)	I (nA)	ΔI (nA)	Date
20.10	0.01	1019	2	18th Nov 05
19.01	0.01	1018	2	20th Nov 05
18.05	0.01	1016	2	20th Nov 05
17.05	0.01	1013	1	20th Nov 05
16.06	0.01	1009	1	20th Nov 05
15.01	0.01	1008	1	20th Nov 05
14.02	0.01	1006	1	20th Nov 05
13.02	0.01	1005	1	20th Nov 05
12.02	0.01	1002	1	20th Nov 05
11.03	0.01	995	1	20th Nov 05
10.00	0.01	981	1	20th Nov 05
9.02	0.01	978	1	20th Nov 05
8.07	0.01	968	1	20th Nov 05
7.03	0.01	958	1	20th Nov 05
6.01	0.01	944	1	20th Nov 05
5.01	0.01	926	1	20th Nov 05
4.04	0.01	906	1	20th Nov 05
3.56	0.01	894	1	20th Nov 05
3.00	0.01	871	1	20th Nov 05
2.501	0.010	849	1	20th Nov 05
2.252	0.010	835	1	20th Nov 05
2.008	0.010	815	1	20th Nov 05
1.753	0.010	796	1	20th Nov 05
1.500	0.010	771	1	20th Nov 05
1.253	0.010	742	0.5	20th Nov 05
1.001	0.010	704	0.5	20th Nov 05
0.871	0.010	680	0.5	20th Nov 05
0.754	0.010	655	0.5	20th Nov 05
0.626	0.010	626	0.5	20th Nov 05
0.501	0.010	593	0.5	20th Nov 05
0.372	0.010	553	1	20th Nov 05
0.252	0.010	513	0.5	20th Nov 05
0.116	0.010	459	0.5	20th Nov 05
0.086	0.010	445	0.5	20th Nov 05
0.000	0.001	391	1	18th Nov 05
-0.077	0.001	345	0.5	20th Nov 05
-0.122	0.001	317	0.5	20th Nov 05
-0.138	0.001	306	0.5	20th Nov 05
-0.158	0.001	292	0.5	20th Nov 05
-0.180	0.001	277	0.5	20th Nov 05
-0.200	0.001	264	1	18th Nov 05
-0.228	0.001	243	1	18th Nov 05
-0.250	0.001	228	1	18th Nov 05
-0.275	0.001	212	0.5	18th Nov 05

-0.301	0.001	196	0.5	18th Nov 05
-0.323	0.001	182	1	18th Nov 05
-0.351	0.001	166	0.5	18th Nov 05
-0.377	0.001	151	0.5	18th Nov 05
-0.400	0.001	138	0.5	18th Nov 05
-0.423	0.001	126	0.5	18th Nov 05
-0.452	0.001	112	0.5	18th Nov 05
-0.473	0.001	102	0.5	18th Nov 05
-0.501	0.001	90.0	0.1	18th Nov 05
-0.524	0.001	80.5	0.1	18th Nov 05
-0.551	0.001	70.3	0.1	18th Nov 05
-0.574	0.001	62.1	0.1	18th Nov 05
-0.602	0.001	53.0	0.05	18th Nov 05
-0.623	0.001	46.5	0.1	18th Nov 05
-0.651	0.001	38.9	0.05	18th Nov 05
-0.674	0.001	33.0	0.05	18th Nov 05
-0.700	0.001	27.2	0.05	18th Nov 05
-0.727	0.001	20.2	0.05	20th Nov 05
-0.749	0.001	16.49	0.02	20th Nov 05
-0.776	0.001	11.91	0.01	20th Nov 05
-0.804	0.001	7.91	0.01	20th Nov 05
-0.826	0.001	5.18	0.01	20th Nov 05
-0.851	0.001	2.36	0.01	20th Nov 05
-0.874	0.001	0.261	0.002	20th Nov 05
-0.907	0.001	-2.18	0.005	20th Nov 05
-0.925	0.001	-3.30	0.005	20th Nov 05
-0.950	0.001	-4.55	0.005	20th Nov 05
-0.975	0.001	-5.58	0.005	20th Nov 05
-1.004	0.001	-6.42	0.01	20th Nov 05
-1.056	0.001	-7.41	0.01	20th Nov 05
-1.103	0.001	-7.91	0.02	20th Nov 05
-1.149	0.001	-8.24	0.01	20th Nov 05
-1.205	0.001	-8.47	0.01	20th Nov 05
-1.307	0.001	-8.67	0.01	20th Nov 05
-1.402	0.001	-8.81	0.01	20th Nov 05
-1.607	0.001	-8.96	0.01	20th Nov 05
-1.803	0.001	-9.07	0.02	20th Nov 05
-2.031	0.001	-9.19	0.01	20th Nov 05
-2.269	0.001	-9.33	0.01	20th Nov 05
-2.497	0.001	-9.37	0.01	20th Nov 05
-2.754	0.001	-9.48	0.01	20th Nov 05
-3.035	0.001	-9.55	0.01	20th Nov 05
-3.52	0.01	-9.69	0.01	20th Nov 05
-4.01	0.01	-9.80	0.01	20th Nov 05
-4.50	0.01	-9.88	0.01	20th Nov 05
-5.01	0.01	-9.98	0.01	20th Nov 05

-5.52	0.01	-10.07	0.01	20th Nov 05
-6.02	0.01	-10.13	0.01	20th Nov 05
-7.00	0.01	-10.26	0.01	20th Nov 05
-8.04	0.01	-10.40	0.01	20th Nov 05
-9.06	0.01	-10.48	0.01	20th Nov 05
-10.02	0.01	-10.55	0.01	20th Nov 05
-11.04	0.01	-10.64	0.01	20th Nov 05
-12.05	0.01	-10.69	0.01	20th Nov 05
-13.03	0.01	-10.75	0.01	20th Nov 05
-14.02	0.01	-10.79	0.01	20th Nov 05
-15.00	0.01	-10.82	0.01	20th Nov 05
-16.08	0.01	-10.82	0.01	20th Nov 05
-17.03	0.01	-10.81	0.01	20th Nov 05
-18.06	0.01	-10.85	0.01	20th Nov 05
-19.01	0.01	-10.91	0.01	20th Nov 05
-20.09	0.01	-10.90	0.01	20th Nov 05

Violet Filter

V (V)	ΔV (V)	I (nA)	ΔI (nA)	Date
20.10	0.01	1019	2	18th Nov 05
19.03	0.01	1015	1	20th Nov 05
18.04	0.01	1011	1	20th Nov 05
17.08	0.01	1007	1	20th Nov 05
16.00	0.01	1002	1	20th Nov 05
15.01	0.01	999	1	20th Nov 05
14.04	0.01	994	1	20th Nov 05
13.05	0.01	991	1	20th Nov 05
12.03	0.01	980	1	20th Nov 05
11.02	0.01	971	1	20th Nov 05
10.03	0.01	964	1	18th Nov 05
9.01	0.01	950	1	20th Nov 05
8.01	0.01	940	1	20th Nov 05
7.03	0.01	926	1	20th Nov 05
6.06	0.01	909	1	20th Nov 05
5.02	0.01	901	1	20th Nov 05
4.50	0.01	887	1	20th Nov 05
4.04	0.01	878	1	20th Nov 05
3.51	0.01	856	1	20th Nov 05
3.04	0.01	836	1	20th Nov 05
2.766	0.001	813	1	20th Nov 05
2.510	0.001	805	1	20th Nov 05
2.237	0.001	782	1	20th Nov 05
2.021	0.001	773	1	20th Nov 05
1.736	0.001	754	1	20th Nov 05
1.503	0.001	729	1	20th Nov 05
1.246	0.001	685	1	20th Nov 05
1.007	0.001	658	1	20th Nov 05
0.873	0.001	632	1	20th Nov 05
0.753	0.001	609	1	20th Nov 05
0.622	0.001	576	1	20th Nov 05
0.502	0.001	548	1	20th Nov 05
0.377	0.001	506	1	20th Nov 05
0.254	0.001	468	1	20th Nov 05
0.175	0.001	436	1	20th Nov 05
0.093	0.001	401	1	20th Nov 05
0.000	0.001	364	1	18th Nov 05
-0.022	0.001	351	1	20th Nov 05
-0.113	0.001	302	0.5	20th Nov 05
-0.152	0.001	280	0.5	20th Nov 05
-0.201	0.001	258	0.5	18th Nov 05
-0.250	0.001	230	1	18th Nov 05
-0.301	0.001	204	0.5	18th Nov 05
-0.352	0.001	179	0.5	18th Nov 05

-0.400	0.001	158	0.5	18th Nov 05
-0.450	0.001	134.0	1	20th Nov 05
-0.501	0.001	113.7	1	20th Nov 05
-0.551	0.001	95.2	0.1	20th Nov 05
-0.601	0.001	78.2	0.1	20th Nov 05
-0.651	0.001	62.9	0.2	20th Nov 05
-0.703	0.001	48.4	0.1	20th Nov 05
-0.751	0.001	36.4	0.05	20th Nov 05
-0.801	0.001	25.0	0.05	20th Nov 05
-0.851	0.001	15.30	0.01	20th Nov 05
-0.900	0.001	8.77	0.01	20th Nov 05
-0.952	0.001	1.585	0.002	20th Nov 05
-1.001	0.001	-3.63	0.005	20th Nov 05
-1.052	0.001	-7.72	0.02	20th Nov 05
-1.102	0.001	-10.70	0.01	20th Nov 05
-1.152	0.001	-12.65	0.01	20th Nov 05
-1.199	0.001	-13.88	0.01	20th Nov 05
-1.251	0.001	-14.77	0.01	20th Nov 05
-1.300	0.001	-15.28	0.01	20th Nov 05
-1.352	0.001	-15.55	0.02	20th Nov 05
-1.404	0.001	-15.78	0.01	20th Nov 05
-1.450	0.001	-15.88	0.01	20th Nov 05
-1.501	0.001	-15.98	0.02	20th Nov 05
-1.551	0.001	-16.06	0.02	20th Nov 05
-1.599	0.001	-16.16	0.01	20th Nov 05
-1.704	0.001	-16.27	0.01	20th Nov 05
-1.806	0.001	-16.39	0.01	20th Nov 05
-1.901	0.001	-16.29	0.01	20th Nov 05
-2.006	0.001	-16.57	0.01	20th Nov 05
-2.253	0.001	-16.77	0.01	20th Nov 05
-2.499	0.001	-16.93	0.02	20th Nov 05
-2.748	0.001	-17.07	0.01	20th Nov 05
-3.004	0.001	-17.21	0.01	20th Nov 05
-3.51	0.01	-17.39	0.01	20th Nov 05
-4.05	0.01	-17.55	0.02	20th Nov 05
-4.51	0.01	-17.70	0.01	20th Nov 05
-5.00	0.01	-17.84	0.01	20th Nov 05
-5.50	0.01	-17.94	0.01	20th Nov 05
-6.00	0.01	-18.02	0.01	20th Nov 05
-7.03	0.01	-18.24	0.01	20th Nov 05
-8.01	0.01	-18.34	0.02	20th Nov 05
-9.02	0.01	-18.43	0.02	20th Nov 05
-10.03	0.01	-18.53	0.02	20th Nov 05
-11.03	0.01	-18.62	0.02	20th Nov 05
-12.05	0.01	-18.70	0.02	20th Nov 05
-13.04	0.01	-18.77	0.01	20th Nov 05

-14.05	0.01	-18.84	0.02	20th Nov 05
-15.03	0.01	-18.88	0.01	20th Nov 05
-16.05	0.01	-18.87	0.01	20th Nov 05
-17.05	0.01	-18.90	0.01	20th Nov 05
-18.00	0.01	-18.93	0.01	20th Nov 05
-19.05	0.01	-18.98	0.01	20th Nov 05
-20.08	0.01	-18.99	0.01	20th Nov 05

UV Filter

V (V)	ΔV (V)	I (nA)	ΔI (nA)	Date
20.10	0.01	1019	2	18th Nov 05
19.05	0.01	1015	2	20th Nov 05
18.06	0.01	1014	2	20th Nov 05
17.03	0.01	1012	2	20th Nov 05
16.07	0.01	1010	2	20th Nov 05
15.02	0.01	1005	2	20th Nov 05
14.03	0.01	1002	2	20th Nov 05
13.02	0.01	998	2	20th Nov 05
12.01	0.01	988	2	20th Nov 05
11.00	0.01	981	1	20th Nov 05
10.02	0.01	971	1	18th Nov 05
9.00	0.01	960	1	20th Nov 05
8.03	0.01	944	1	20th Nov 05
7.02	0.01	926	1	20th Nov 05
6.01	0.01	909	1	20th Nov 05
5.03	0.01	884	2	18th Nov 05
4.51	0.01	872	1	20th Nov 05
4.02	0.01	854	1	20th Nov 05
3.51	0.01	837	1	20th Nov 05
3.01	0.01	810	1	20th Nov 05
3.49	0.01	835	1	20th Nov 05
3.22	0.01	819	1	20th Nov 05
3.00	0.01	810	1	20th Nov 05
2.754	0.001	795	1	20th Nov 05
2.506	0.001	782	1	20th Nov 05
2.376	0.001	774	1	20th Nov 05
2.254	0.001	764	1	20th Nov 05
2.124	0.001	756	1	20th Nov 05
2.002	0.001	745	1	20th Nov 05
1.875	0.001	733	1	20th Nov 05
1.755	0.001	720	1	20th Nov 05
1.625	0.001	708	1	20th Nov 05
1.499	0.001	694	1	20th Nov 05
1.377	0.001	679	1	20th Nov 05
1.256	0.001	664	1	20th Nov 05
1.121	0.001	645	1	20th Nov 05
1.006	0.001	626	1	20th Nov 05
0.874	0.001	601	1	20th Nov 05
0.749	0.001	575	1	20th Nov 05
0.621	0.001	547	1	20th Nov 05
0.504	0.001	517	1	20th Nov 05
0.376	0.001	482	1	20th Nov 05
0.253	0.001	446	1	20th Nov 05
0.132	0.001	404	1	20th Nov 05

0.074	0.001	381	1	20th Nov 05
0.000	0.001	353	1	20th Nov 05
-0.021	0.001	343	1	20th Nov 05
-0.046	0.001	333	1	20th Nov 05
-0.075	0.001	320	1	20th Nov 05
-0.094	0.001	311	1	20th Nov 05
-0.112	0.001	302	1	20th Nov 05
-0.135	0.001	290	1	20th Nov 05
-0.169	0.001	275	1	20th Nov 05
-0.200	0.001	260	0.5	20th Nov 05
-0.248	0.001	234	0.5	18th Nov 05
-0.303	0.001	210	0.5	18th Nov 05
-0.351	0.001	190	0.5	18th Nov 05
-0.404	0.001	170	0.5	18th Nov 05
-0.448	0.001	155	1	18th Nov 05
-0.502	0.001	138	0.5	18th Nov 05
-0.551	0.001	124	0.5	18th Nov 05
-0.604	0.001	109.2	0.1	18th Nov 05
-0.648	0.001	98.0	0.1	18th Nov 05
-0.705	0.001	81.9	0.1	20th Nov 05
-0.756	0.001	69.2	0.1	20th Nov 05
-0.805	0.001	58.1	0.1	20th Nov 05
-0.853	0.001	47.9	0.1	20th Nov 05
-0.905	0.001	37.4	0.1	20th Nov 05
-0.948	0.001	29.3	0.1	20th Nov 05
-1.002	0.001	20.4	0.05	20th Nov 05
-1.051	0.001	13.2	0.05	20th Nov 05
-1.103	0.001	7.00	0.01	20th Nov 05
-1.152	0.001	1.960	0.002	20th Nov 05
-1.202	0.001	-2.24	0.05	20th Nov 05
-1.250	0.001	-5.47	0.02	20th Nov 05
-1.302	0.001	-8.10	0.02	20th Nov 05
-1.350	0.001	-9.94	0.01	20th Nov 05
-1.406	0.001	-11.44	0.01	20th Nov 05
-1.454	0.001	-12.29	0.01	20th Nov 05
-1.500	0.001	-12.87	0.01	20th Nov 05
-1.553	0.001	-13.28	0.01	20th Nov 05
-1.600	0.001	-13.50	0.01	20th Nov 05
-1.651	0.001	-13.70	0.02	20th Nov 05
-1.703	0.001	-13.80	0.02	20th Nov 05
-1.752	0.001	-13.85	0.02	20th Nov 05
-1.801	0.001	-13.92	0.02	20th Nov 05
-1.852	0.001	-13.98	0.02	20th Nov 05
-1.904	0.001	-14.03	0.02	20th Nov 05
-1.945	0.001	-14.08	0.01	20th Nov 05
-2.003	0.001	-14.13	0.01	20th Nov 05
-2.104	0.001	-14.20	0.01	20th Nov 05

-2.203	0.001	-14.31	0.01	20th Nov 05
-2.303	0.001	-14.35	0.01	20th Nov 05
-2.404	0.001	-14.42	0.01	20th Nov 05
-2.505	0.001	-14.46	0.01	20th Nov 05
-2.758	0.001	-14.58	0.01	20th Nov 05
-3.006	0.001	-14.70	0.01	20th Nov 05
-3.24	0.01	-14.77	0.01	20th Nov 05
-3.53	0.01	-14.85	0.01	20th Nov 05
-3.78	0.01	-14.90	0.01	20th Nov 05
-4.06	0.01	-14.97	0.01	20th Nov 05
-4.51	0.01	-15.09	0.01	20th Nov 05
-5.05	0.01	-15.14	0.01	20th Nov 05
-5.54	0.01	-15.23	0.02	20th Nov 05
-6.07	0.01	-15.30	0.02	20th Nov 05
-6.52	0.01	-15.33	0.02	20th Nov 05
-7.04	0.01	-15.39	0.02	20th Nov 05
-8.03	0.01	-15.46	0.01	20th Nov 05
-9.03	0.01	-15.64	0.02	20th Nov 05
-10.06	0.01	-15.65	0.01	20th Nov 05
-11.02	0.01	-15.75	0.02	20th Nov 05
-12.04	0.01	-15.75	0.02	20th Nov 05
-13.02	0.01	-15.80	0.02	20th Nov 05
-14.07	0.01	-15.84	0.02	20th Nov 05
-15.04	0.01	-15.87	0.02	20th Nov 05
-16.03	0.01	-15.88	0.02	20th Nov 05
-17.04	0.01	-15.91	0.02	20th Nov 05
-18.00	0.01	-15.92	0.02	20th Nov 05
-19.06	0.01	-15.95	0.02	20th Nov 05
-20.08	0.01	-15.99	0.01	20th Nov 05

Dark Current (Rainy day, with lights off)

V (V)	ΔV (V)	I (nA)	ΔI (nA)
20.15	0.01	0.627	0.004
19.03	0.01	0.570	0.004
18.04	0.01	0.551	0.003
17.01	0.01	0.515	0.002
16.04	0.01	0.500	0.004
15.07	0.01	0.453	0.002
14.01	0.01	0.420	0.002
13.03	0.01	0.400	0.002
12.04	0.01	0.360	0.002
11.02	0.01	0.333	0.004
9.99	0.01	0.300	0.004
9.01	0.01	0.280	0.004
8.00	0.01	0.240	0.005
7.05	0.01	0.216	0.004
6.07	0.01	0.182	0.004
5.07	0.01	0.149	0.004
4.00	0.01	0.120	0.004
3.04	0.01	0.094	0.003
2.002	0.001	0.064	0.004
1.015	0.001	0.030	0.002
0.000	0.001	0.000	0.001
-1.002	0.001	-0.069	0.002
-2.033	0.001	-0.144	0.002
-3.08	0.01	-0.216	0.002
-4.07	0.01	-0.275	0.002
-5.00	0.01	-0.336	0.003
-6.03	0.01	-0.400	0.003
-7.02	0.01	-0.469	0.002
-8.05	0.01	-0.539	0.003
-9.04	0.01	-0.583	0.003
-10.06	0.01	-0.632	0.003
-11.00	0.01	-0.682	0.003
-11.99	0.01	-0.720	0.003
-13.06	0.01	-0.754	0.002
-14.02	0.01	-0.789	0.002
-15.10	0.01	-0.840	0.002
-16.05	0.01	-0.870	0.003
-17.00	0.01	-0.897	0.004
-18.01	0.01	-0.920	0.004
-19.00	0.01	-0.947	0.004
-20.10	0.01	-0.979	0.003

Appendix C: Matlab Code for Data Analysis

lab03.m

```
% Lab E-10

filters{1} = struct('Name', 'yellow', 'Wavelength', 5780,...
    'Color', [.9,.8,0], 'Marker', 'square');
filters{2} = struct('Name', 'green', 'Wavelength', 5461,...
    'Color', [0,1,0], 'Marker', 'diamond');
filters{3} = struct('Name', 'blue', 'Wavelength', 4358,...
    'Color', [0,0,1], 'Marker', 'o');
filters{4} = struct('Name', 'violet', 'Wavelength', 4047,...
    'Color', [.77,0,.9], 'Marker', 'v');
filters{5} = struct('Name', 'uv', 'Wavelength', 3663,...
    'Color', [.85,0,0], 'Marker', '^');
fcount = length(filters);

%-------------------------%-------------------------------------------------
% Background Dark Current %
%-------------------------%

dats{1} = load('../data2/bg1.txt');
dats{2} = load('../data2/bg2.txt'); % worst case of dark current

figure; hold on; grid on;
for i = length(dats)
    dat = dats{i};
    V = dat(:,1);
    I = dat(:,2);
    dI = dat(:,3); % add drift error
    Pbg = polyfit(V,I, 1);
        plot(V,I, '.r');
        drawErrorBars(V,I, dI, .3,...
                      [1,.1,.1]);
        plot(V,polyval(Pbg,V), '-b');
end
legend('cloudy day, lights on');
ylabel('I (nA)');
xlabel('V (V)');
title('Dark Current');

%-------------------%-----------------------------------------------------
% Calibration Drift %
%-------------------%

% this is a very very rough estimate,
% because of real dependence on the time to take measurements
% given in absolute value of drift of I0
filters{1}.Drift = 1 *1019/100.5; % worst; note smallest I0
filters{2}.Drift = 3;
filters{3}.Drift = 1; % best
filters{4}.Drift = 4; % bad
filters{5}.Drift = 3.5; % bad

%-----------%-------------------------------------------------------------
```

```
% Load Data %
%-----------%

for i = 1:fcount
    dat = load(['../data3/' filters{i}.Name '.txt']);

    filters{i}.V = dat(:,1);
    filters{i}.I = dat(:,3);
    filters{i}.dV = dat(:,2);
    filters{i}.dI = dat(:,4) + abs(polyval(Pbg,filters{i}.dV))...
        + filters{i}.Drift;

    I0 = filters{i}.I(1);
    dI0 = filters{i}.dI(1);
    filters{i}.J = filters{i}.I ./ I0; % normalize
    filters{i}.dJ = 1/I0 * sqrt(filters{i}.dI.^2 ...
        + (filters{i}.I.*(dI0/I0)).^2);
    filters{i}.W = (1./filters{i}.dJ).^2;
end

%--------------%-------------------------------------------------------------
% General Plot %
%--------------%

figure; grid on; hold on;
legh_ = []; legt_ = {};   % handles and text for legend
for i = 1:fcount
    h_ = plot(filters{i}.V,filters{i}.I, 'x', 'Color', filters{i}.Color,...
        'Marker', filters{i}.Marker);
    drawErrorBars(filters{i}.V,filters{i}.I, filters{i}.dI, .2,...
                  filters{i}.Color);

    legh_(end+1) = h_(1);
    legt_{end+1} = filters{i}.Name;
end
ylabel('I (nA)');
xlabel('V (V)');
legend(legh_, legt_);
title('I-V Curves at Different Frequencies');

%-----------------------------%-------------------------------------------
% Reverse Normalized Currents %
%-----------------------------%

for i = 1:fcount
    rrange = find(filters{i}.V <= -12); % for linear fits
    filters{i}.rrange = rrange;
    filters{i}.Vr = filters{i}.V(rrange);
    filters{i}.Jr = filters{i}.J(rrange);
    filters{i}.dVr = filters{i}.dV(rrange);
    filters{i}.dJr = filters{i}.dJ(rrange);

    [filters{i}.Jr_fit, filters{i}.dJr_fit] = ...
        linearfit(filters{i}.Vr,filters{i}.Jr,...
                  filters{i}.dVr,filters{i}.dJr,...
                  filters{i}.V,filters{i}.dV);
    % allow only non-positive values
```

```
    filters{i}.Jr_fit(filters{i}.Jr_fit > 0) = 0;
end
figure;
subplot(2,1,1); grid on; hold on;
    for i = 1:fcount
        plot(filters{i}.Vr, filters{i}.Jr, 'x',...
            'Marker', filters{i}.Marker, 'Color', filters{i}.Color);
        drawErrorBars(filters{i}.Vr,filters{i}.Jr, filters{i}.dJr, .2,...
                      filters{i}.Color);
        plot(filters{i}.Vr, filters{i}.Jr_fit(filters{i}.rrange),'-',...
            'Color', filters{i}.Color);
    end
    ylabel('J');
    xlabel('V (V)');
    title('Reverse Normalized Currents (J = I/I_0)');
subplot(2,1,2); grid on; hold on;
    for i = 1:fcount
        res = filters{i}.Jr - filters{i}.Jr_fit(filters{i}.rrange);
        plot(filters{i}.Vr, res, 'x',...
            'Marker', filters{i}.Marker, 'Color', filters{i}.Color);
    end
    ylabel('J');
    xlabel('V (V)');
    title('Residuals');

%-----------------------------%-------------------------------------------
% Investigation of Derivative %
%-----------------------------%

figure; grid on; hold on;
for i = 1:fcount
    % numerical differentiation, approximate
    D = diff(filters{i}.J - filters{i}.Jr_fit) ./ diff(filters{i}.V);
    filters{i}.DJJr_DV = D;
    plot(filters{i}.V(2:end), D,...
        'Marker', filters{i}.Marker, 'Color', filters{i}.Color);
end

%----------------%-----------------------------------------------------------
% Intercept Fits %
%----------------%

Vfitub = [-.24   -.18     -.2     -.15     -.25];
Vfitlb = [-.31   -.38     -.6     -.8      -1.1];
for i = 1:fcount
    irange = find(filters{i}.V(find(filters{i}.V >= Vfitlb(i)))...
                  <= Vfitub(i));

    K = sqrt(filters{i}.J - filters{i}.Jr_fit);
    dK = 1./K .* sqrt(filters{i}.dJ.^2 + filters{i}.dJr_fit.^2);
    filters{i}.K = K(irange); % reduce range
    filters{i}.dK = dK(irange);
    filters{i}.Vi = filters{i}.V(irange);
    filters{i}.dVi = filters{i}.dV(irange);

    [filters{i}.K_fit, filters{i}.dK_fit, A,B, dA,dB] = ...
        linearfit(filters{i}.Vi,filters{i}.K,...
```

```
                  filters{i}.dVi,filters{i}.dK,...
                  filters{i}.Vi,filters{i}.dVi);

    filters{i}.Vs = --A/B;
    filters{i}.dVs = sqrt((A/B^2*dB)^2 + (-dA/B)^2);
end
figure;
subplot(2,1,1); grid on; hold on;
    for i = 1:fcount
        plot(filters{i}.Vi, filters{i}.K, 'x',...
            'Marker', filters{i}.Marker, 'Color', filters{i}.Color);
        drawErrorBars(filters{i}.Vi,filters{i}.K, filters{i}.dK, .02,...
                      filters{i}.Color);
        plot(filters{i}.Vi, filters{i}.K_fit,'-',...
            'Color', filters{i}.Color);
    end
    ylabel('K');
    xlabel('V (V)');
    title('Intercept Region of Transformed Currents (K = J - J_r)');
subplot(2,1,2); grid on; hold on;
    for i = 1:fcount
        res = filters{i}.K - filters{i}.K_fit;
        plot(filters{i}.Vi, res, 'x',...
            'Marker', filters{i}.Marker, 'Color', filters{i}.Color);
    end
    ylabel('K');
    xlabel('V (V)');
    title('Residuals');

%-------------------%-------------------------------------------------------
% Stopping Voltages %
%-------------------%

% order: yellow, green, blue, violet, uv
c = 299792458; % speed of light, in metres
Vs = [filters{1}.Vs filters{2}.Vs filters{3}.Vs filters{4}.Vs...
     filters{5}.Vs];
dVs = [filters{1}.dVs filters{2}.dVs filters{3}.dVs filters{4}.dVs...
      filters{5}.dVs];
L = [filters{1}.Wavelength filters{2}.Wavelength filters{3}.Wavelength...
    filters{4}.Wavelength filters{5}.Wavelength] * 1e-10; % wavelength in m
F = c ./ L; % frequency
f = F(1):(F(end)-F(1))/20:F(end);

P = polyfit(F,Vs,1);

figure; grid on; hold on;
plot(F,Vs, 'xr');
drawErrorBars(F,Vs, dVs, .1e14, filters{i}.Color);
plot(f, polyval(P, f), '-r');

e = 1.60217646e-19; % electron charge
P1 = P;
h = P(1) * e

w = 1./dVs.^2;
DELTA = sum(w)*(w*(F.^2)') - (w*F')^2;
```

```
dB = sqrt(sum(w) / DELTA);
dh = e * dB;


% compare against h = 6.626068e-34 m^2 kg/s
h0 = 6.626068e-34;
err = abs(h-h0) / h0 % deviation

% lab manual paper
L = [5770 5461 4916 4339 3650] * 1e-10;
Vs = [0.52 0.59 0.83 1.14 1.72];
F = c ./ L; % frequency
f = F(1):(F(end)-F(1))/20:F(end);
P = polyfit(F,Vs,1);
plot(F,Vs, 'oc');
plot(f, polyval(P, f), '-c');


ylabel('Stopping Voltage (V)');
xlabel('Freq (Hz)');
titlestr = 'Stopping Voltage at Different Frequencies';
titlestr = sprintf([titlestr '\n' 'h = ' num2str(h*1e34) ' +/- '...
    num2str(dh*1e34) ' * 10^{-34} {m^2}kg/s']);
titlestr = sprintf([titlestr '\n' num2str(err*100) '%% from h_0 = ' num2str(h0)]);
title(titlestr);
```

---

linearfit.m

```
function [y,dy, A,B, dA,dB] = linearfit(X,Y,dX,dY, x,dx)

    P = polyfit(X,Y, 1);

    A = P(2);
    B = P(1);

    % change to equivalent error in Y
    dY = sqrt( dY.^2 + (B.*dX).^2 );
    dX = 0;

    w = 1./dY.^2; % weights

    DELTA = sum(w)*(w'*X.^2) - (w'*X)^2;
    dA = sqrt( w'*X.^2 / DELTA );
    dB = sqrt( sum(w) / DELTA );

    y = polyval(P,x);
    dy = sqrt( dA^2 + (x.*dB).^2 + (B.*dx).^2 );

end
```